Solution by the PROPOSER.

The only solution x divisible by p is seen to be $\equiv 0(\text{mod} p^n)$. Let next x be prime to p. Then

$$x^{p^n-1}\equiv 1(\text{mod} p^n),\quad x^{p^{n-2}(p-1)}\equiv 1(\text{mod} p^n),$$

the second from Fermat's theorem generalized, since $\phi(p^n)=p^{n-1}(p-1)$. Hence must $x^{p-1}\equiv 1(\text{mod} p^n)$. To prove that the latter has precisely $p-1$ distinct solutions modp^n, we proceed by induction, the result being true for $n=1$ by Fermat's theorem. Let therefore $x_1, \ldots, x_{p-1}$ be the distinct roots of $x^{p-1}\equiv 1(\text{mod} p^a)$, whence $x_i{}^p=x_i+r_ip^a$, each r_i being a fixed integer. To determine the roots of $x^{p-1}\equiv 1(\text{mod} p^{a+1})$, we seek the integral values of m_i incongruent mod$p$ such that $x=x_i+m_ip^a$ satifies $x^p\equiv x(\text{mod} p^{a+1})$. But $(x_i+m_ip^a)^p=x_i{}^p+$multiple of p^{a+1}, by the Binomial Theorem. The condition is therefore

$$x_i+r_ip^a\equiv x_i+m_ip^a\ (\text{mod} p^{a+1}),$$

whence $m_i\equiv r_i(\text{mod} p)$ and m_i is uniquely determined modulo p.

AVERAGE AND PROBABILITY.

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148. Proposed by M. C. RORTY, Boston, Mass.

Assuming n points to fall at random upon a circle of circumference a, what is the probability of m or more points falling within a length b upon this circumference?

NOTE. This problem has practical application in determining the probability of accidental rushes of telephone calls as distinct from those rushes which are due to commercial causes. The solution for m or more points falling within a specified length b is known. The problem presented above differs from this in that a solution is required for *any* length b.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $m+r$, $(r=0, 1, 2, 3, \ldots n-m)$ be the number of points to fall on b. $m+r$ points can be selected from n points in $n!\div(m+r)!(n-m-r)!$ ways.

$$\therefore \text{Chance}=\frac{2n!}{(m+r)!(n-m-r)!}\cdot\frac{(b)^{m+r}(a-b)^{n-m-r}}{a^n}.$$

149. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Three points are taken at random on the convex surface of a right cone. Find the probability that the section of the cone made by the plane passing through them is a complete ellipse.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let ABC be a section of the cone, E one of the random points in the surface. $AD=c$, $GD=h$, $BD=R$, $EG=r$. Through E pass the plane EC. Then the area of the surface, of which EBC is a projection

$$=\frac{\pi\sqrt{[h^2+(R-r)^2]}}{R-r}[R^2-(R+r)\sqrt{(Rr)}].$$